UFRGS – INSTITUTO DE MATEMÁTICA
Departamento de Matemática Pura e Aplicada
MAT01168 - Turma A - 2018/1
Prova da área IIB

1 - 6	7	8	Total

Nome: ______________________________ **Cartão:** ______________

Regras Gerais:

- Não é permitido o uso de calculadoras, telefones ou qualquer outro recurso computacional ou de comunicação.
- Trabalhe individualmente e sem uso de material de consulta além do fornecido.
- Devolva o caderno de questões preenchido ao final da prova.

Regras para as questões abertas:

- Seja sucinto, completo e claro.
- Justifique todo procedimento usado.
- Indique identidades matemáticas usadas, em especial, itens da tabela.
- Use notação matemática consistente!

Propriedades das transformadas de Fourier: considere a notação $F(w) = \mathcal{F}\{f(t)\}$.

1.	Linearidade	$\mathcal{F}\{\alpha f(t) + \beta g(t)\} = \alpha\mathcal{F}\{f(t)\} + \beta\mathcal{F}\{g(t)\}$
2.	Transformada da derivada	Se $\lim_{t\to\pm\infty} f(t) = 0$, então $\mathcal{F}\{f'(t)\} = iw\mathcal{F}\{f(t)\}$ Se $\lim_{t\to\pm\infty} f(t) = \lim_{t\to\pm\infty} f'(t) = 0$, então $\mathcal{F}\{f''(t)\} = -w^2\mathcal{F}\{f(t)\}$
3.	Deslocamento no eixo w	$\mathcal{F}\{e^{at}f(t)\} = F(w + ia)$
4.	Deslocamento no eixo t	$\mathcal{F}\{f(t-a)\} = e^{-iaw}F(w)$
5.	Transformada da integral	Se $F(0) = 0$, então $\mathcal{F}\left\{\int_{-\infty}^{t} f(\tau)d\tau\right\} = \frac{F(w)}{iw}$
6.	Teorema da modulação	$\mathcal{F}\{f(t)\cos(w_0t)\} = \frac{1}{2}F(w-w_0) + \frac{1}{2}F(w+w_0)$
7.	Teorema da Convolução	$\mathcal{F}\{(f*g)(t)\} = F(w)G(w)$, onde $(f*g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$ $(F*G)(w) = 2\pi\mathcal{F}\{f(t)g(t)\}$
8.	Conjugação	$\overline{F(w)} = F(-w)$
9.	Inversão temporal	$\mathcal{F}\{f(-t)\} = F(-w)$
10.	Simetria ou dualidade	$f(-w) = \frac{1}{2\pi}\mathcal{F}\{F(t)\}$
11.	Mudança de escala	$\mathcal{F}\{f(at)\} = \frac{1}{\lvert a\rvert}F\left(\frac{w}{a}\right), \quad a \neq 0$
12.	Teorema da Parseval	$\int_{-\infty}^{\infty} \lvert f(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert F(w)\rvert^2 dw$
13.	Teorema da Parseval para Série de Fourier	$\frac{1}{T}\int_0^T \lvert f(t)\rvert^2 dt = \sum_{n=-\infty}^{\infty} \lvert C_n\rvert^2$

Séries e transformadas de Fourier:

	Forma trigonométrica	Forma exponencial
Série de Fourier	$f(t) = \frac{a_0}{2} + \sum_{n=1}^{N}[a_n\cos(w_nt) + b_n\,\text{sen}(w_nt)]$ onde $w_n = \frac{2\pi n}{T}$, T é o período de $f(t)$ $a_0 = \frac{2}{T}\int_0^T f(t)dt = \frac{2}{T}\int_{-T/2}^{T/2} f(t)dt,$ $a_n = \frac{2}{T}\int_0^T f(t)\cos(w_nt)dt = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(w_nt)dt,$ $b_n = \frac{2}{T}\int_0^T f(t)\,\text{sen}(w_nt)dt = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\,\text{sen}(w_nt)dt$	$f(t) = \sum_{n=-\infty}^{\infty} C_ne^{iw_nt},$ onde $C_n = \frac{a_n - ib_n}{2}$
Transformada de Fourier	$f(t) = \frac{1}{\pi}\int_0^{\infty}(A(w)\cos(wt) + B(w)\,\text{sen}(wt))\,dw$, para $f(t)$ real, onde $A(w) = \int_{-\infty}^{\infty} f(t)\cos(wt)dt$ e $B(w) = \int_{-\infty}^{\infty} f(t)\,\text{sen}(wt)dt$	$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)e^{iwt}dw,$ onde $F(w) = \int_{-\infty}^{\infty} f(t)e^{-iwt}dt$

Tabela de integrais definidas:

1. $\int_0^\infty e^{-ax}\cos(mx)dx = \frac{a}{a^2+m^2} \quad (a>0)$	2. $\int_0^\infty e^{-ax}\operatorname{sen}(mx)dx = \frac{m}{a^2+m^2} \quad (a>0)$
3. $\int_0^\infty \frac{\cos(mx)}{a^2+x^2}dx = \frac{\pi}{2a}e^{-ma} \quad (a>0,\ m\geq 0)$	4. $\int_0^\infty \frac{x\operatorname{sen}(mx)}{a^2+x^2}dx = \frac{\pi}{2}e^{-ma} \quad (a\geq 0,\ m>0)$
5. $\int_0^\infty \frac{\operatorname{sen}(mx)\cos(nx)}{x}dx = \begin{cases} \frac{\pi}{2}, & n<m \\ \frac{\pi}{4}, & n=m, \\ 0, & n>m \end{cases} \quad (m>0,\ n>0)$	6. $\int_0^\infty \frac{\operatorname{sen}(mx)}{x}dx = \begin{cases} \frac{\pi}{2}, & m>0 \\ 0, & m=0 \\ -\frac{\pi}{2}, & m<0 \end{cases}$
7. $\int_0^\infty e^{-r^2x^2}dx = \frac{\sqrt{\pi}}{2r} \quad (r>0)$	8. $\int_0^\infty e^{-a^2x^2}\cos(mx)dx = \frac{\sqrt{\pi}}{2a}e^{-\frac{m^2}{4a^2}} \quad (a>0)$
9. $\int_0^\infty xe^{-ax}\operatorname{sen}(mx)dx = \frac{2am}{(a^2+m^2)^2} \quad (a>0)$	10. $\int_0^\infty e^{-ax}\operatorname{sen}(mx)\cos(nx)dx = = \frac{m(a^2+m^2-n^2)}{(a^2+(m-n)^2)(a^2+(m+n)^2)} \quad (a>0)$
11. $\int_0^\infty xe^{-ax}\cos(mx)dx = \frac{a^2-m^2}{(a^2+m^2)^2} \quad (a>0)$	12. $\int_0^\infty \frac{\cos(mx)}{x^4+4a^4}dx = \frac{\pi}{8a^3}e^{-ma}(\operatorname{sen}(ma)+\cos(ma))$
13. $\int_0^\infty \frac{\operatorname{sen}^2(mx)}{x^2}dx = \lvert m\rvert\frac{\pi}{2}$	14. $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-z^2}dz$
15. $\int_0^\infty \frac{\operatorname{sen}^2(ax)\operatorname{sen}(mx)}{x}dx = \begin{cases} \frac{\pi}{4}, & (0<m<2a) \\ \frac{\pi}{8}, & (0<2a=m) \\ 0, & (0<2a<m) \end{cases}$	16. $\int_0^\infty \frac{\operatorname{sen}(mx)\operatorname{sen}(nx)}{x^2}dx = \begin{cases} \frac{\pi m}{2}, & (0<m\leq n) \\ \frac{\pi n}{2}, & (0<n\leq m) \end{cases}$
17. $\int_0^\infty x^2e^{-ax}\operatorname{sen}(mx)dx = \frac{2m(3a^2-m^2)}{(a^2+m^2)^3} \quad (a>0)$	18. $\int_0^\infty x^2e^{-ax}\cos(mx)dx = \frac{2a(a^2-3m^2)}{(a^2+m^2)^3} \quad (a>0)$
19. $\int_0^\infty \frac{\cos(mx)}{(a^2+x^2)^2}dx = \frac{\pi}{4a^3}(1+ma)e^{-ma} \quad (a>0,\ m\geq 0)$	20. $\int_0^\infty \frac{x\operatorname{sen}(mx)}{(a^2+x^2)^2}dx = \frac{\pi m}{4a}e^{-ma} \quad (a>0,\ m>0)$
21. $\int_0^\infty \frac{x^2\cos(mx)}{(a^2+x^2)^2}dx = \frac{\pi}{4a}(1-ma)e^{-ma} \quad (a>0,\ m\geq 0)$	22. $\int_0^\infty xe^{-a^2x^2}\operatorname{sen}(mx)dx = \frac{m\sqrt{\pi}}{4a^3}e^{-\frac{m^2}{4a^2}} \quad (a>0)$

Frequências das notas musicais em Hertz:

Nota \ Escala	1	2	3	4	5	6
Dó	65,41	130,8	261,6	523,3	1047	2093
Dó ♯	69,30	138,6	277,2	554,4	1109	2217
Ré	73,42	146,8	293,7	587,3	1175	2349
Ré ♯	77,78	155,6	311,1	622,3	1245	2489
Mi	82,41	164,8	329,6	659,3	1319	2637
Fá	87,31	174,6	349,2	698,5	1397	2794
Fá ♯	92,50	185,0	370,0	740,0	1480	2960
Sol	98,00	196,0	392,0	784,0	1568	3136
Sol ♯	103,8	207,7	415,3	830,6	1661	3322
Lá	110,0	220,0	440,0	880,0	1760	3520
Lá ♯	116,5	233,1	466,2	932,3	1865	3729
Si	123,5	246,9	493,9	987,8	1976	3951

Identidades Trigonométricas:

$\cos(x)\cos(y) = \frac{\cos(x+y)+\cos(x-y)}{2}$
$\operatorname{sen}(x)\operatorname{sen}(y) = \frac{\cos(x-y)-\cos(x+y)}{2}$
$\operatorname{sen}(x)\cos(y) = \frac{\operatorname{sen}(x+y)+\operatorname{sen}(x-y)}{2}$

Integrais:

$\int xe^{\lambda x}\ \mathrm{d}x = \frac{e^{\lambda x}}{\lambda^2}(\lambda x-1)+C$
$\int x^2e^{\lambda x}\ \mathrm{d}x = e^{\lambda x}\left(\frac{x^2}{\lambda}-\frac{2x}{\lambda^2}+\frac{2}{\lambda^3}\right)+C$
$\int x^ne^{\lambda x}\ \mathrm{d}x = \frac{1}{\lambda}x^ne^{\lambda x}-\frac{n}{\lambda}\int x^{n-1}e^{\lambda x}\mathrm{d}x+C$
$\int x\cos(\lambda x)\,dx = \frac{\cos(\lambda x)+\lambda x\operatorname{sen}(\lambda x)}{\lambda^2}+C$
$\int x\operatorname{sen}(\lambda x)\,dx = \frac{\operatorname{sen}(\lambda x)-\lambda x\cos(\lambda x)}{\lambda^2}+C$

• **Questão 1** (1.0 ponto) Considere a função $f(t) = 8\cos^4(t)$. Calcule os coeficientes da expansão em série de Fourier de $f(t)$ e assinale na primeira coluna a representação trigonométrica e na segunda a representação exponencial.

( ) $3 + 8\sum_{n=1}^{\infty}\left(\frac{1}{2n+1}\cos(2nt) + \frac{n}{2n+1}\operatorname{sen}(2nt)\right)$

( ) $3 + \sum_{n=1}^{\infty}\frac{1}{2n}e^{2int}$

( ) $3 + 4\cos(t) + 2\cos(2t) + \cos(3t) + \frac{1}{2}\cos(4t)$

(X) $3 + 4\cos(2t) + \cos(4t)$

( ) $3 + 4\operatorname{sen}(t) + 2\operatorname{sen}(2t)$

( ) $\sum_{n=-\infty}^{\infty}\left(\frac{3}{2n+1} - \frac{in}{2n^2+1}\right)e^{2nit}$

( ) $3 + \sum_{n=1}^{\infty}\frac{1}{2n}\cos(2nt)$

( ) $\frac{i}{2}e^{-4it} + 2e^{-2it} + 3 + 2e^{2it} - \frac{i}{2}e^{4it}$

(X) $\frac{1}{2}e^{-4it} + 2e^{-2it} + 3 + 2e^{2it} + \frac{1}{2}e^{4it}$

( ) $\frac{i}{2}e^{-2it} + 2ie^{-it} + 3 - 2ie^{it} - \frac{i}{2}e^{2it}$

• **Questão 2** (1.0 ponto) Considere a função periódica $f(t) = \operatorname{sen}(4t) + \operatorname{sen}(6t) + \operatorname{sen}(8t)$. Marque na primeira coluna o período fundamental e, na segunda, a frequência angular fundamental.

( ) 1

( ) 2

( ) 4

(X) π

( ) 2π

( ) 4π

( ) 1

(X) 2

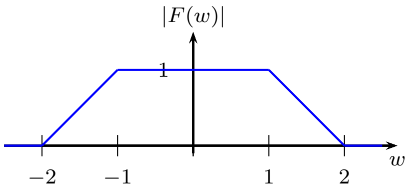
( ) 4

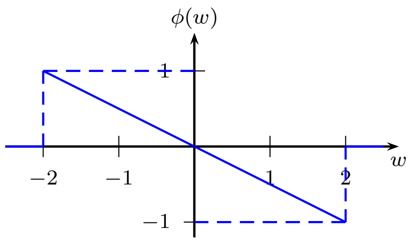
( ) π

( ) 2π

( ) 4π

• **Questão 3** (1.0 ponto) Considere os diagramas de espetro de magnitude e de fase da transformada de Fourier $F(w) = \mathcal{F}\{f(t)\}$ dados nos gráficos abaixo:





Assinale na primeira coluna $F(w)$ e, na segunda, a correta afirmação sobre $f(t)$.

( ) $F(w) = \begin{cases} 1, & -1 \le w \le 1 \\ w+2, & w < -1 \\ 2-w, & w > 1 \end{cases}$

( ) $F(w) = \begin{cases} 1, & -1 \le w \le 1 \\ w+2, & -2 < w < -1 \\ 2-w, & 1 < w < 2 \\ 0, & |w| \ge 2 \end{cases}$

(X) $F(w) = \begin{cases} e^{-iw/2}, & -1 \le w \le 1 \\ (w+2)e^{-iw/2}, & -2 < w < -1 \\ (2-w)e^{-iw/2}, & 1 < w < 2 \\ 0, & |w| \ge 2 \end{cases}$

( ) $F(w) = \begin{cases} e^{-iw}, & -1 \le w \le 1 \\ (w+2)e^{-iw}, & -2 < w < -1 \\ (2-w)e^{-iw}, & 1 < w < 2 \\ 0, & |w| \ge 2 \end{cases}$

( ) Não há suficiente informação para conhecer $F(w)$

(X) $f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)e^{iwt}dw$

( ) $f(t) = \int_{-\infty}^{\infty} F(w)e^{-iwt}dw$

( ) $f(t) = \begin{cases} e^{-it/2}, & -1 \le t \le 1 \\ (t+2)e^{-it/2}, & -2 < t < -1 \\ (2-t)e^{-it/2}, & 1 < t < 2 \\ 0, & |t| \ge 2 \end{cases}$

( ) $f(t) = \begin{cases} e^{-it}, & -1 \le t \le 1 \\ (t+2)e^{-it}, & -2 < t < -1 \\ (2-t)e^{-it}, & 1 < t < 2. \end{cases}$

( ) Não há suficiente informação para conhecer $f(t)$

• **Questão 4** (1.0 pontos) Considere a função $f(t) = 1 + \text{sen}(2\pi t) + \cos(2\pi t) + \text{sen}(4\pi t)$. Assinale na primeira coluna o diagrama de espectro do módulo e, na segunda, o diagrama de espectro da fase.

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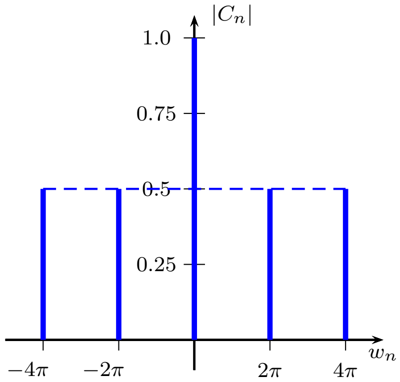

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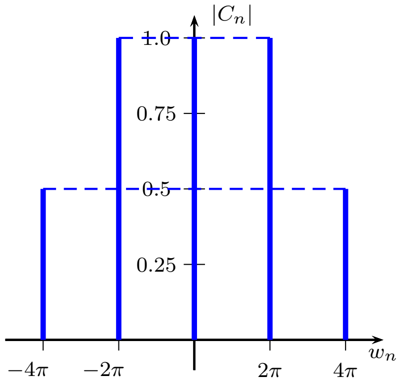

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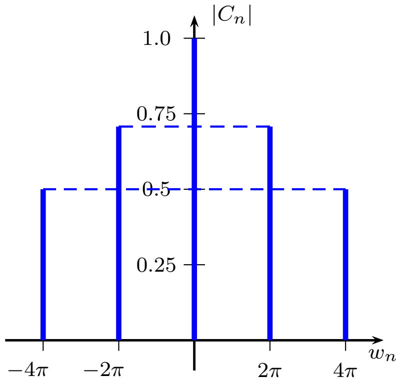

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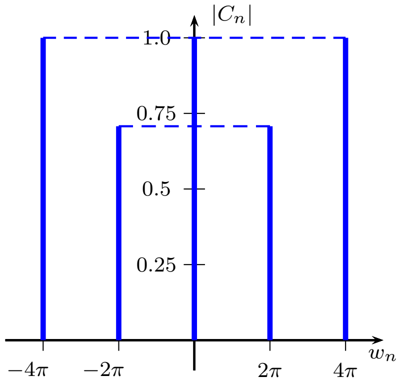

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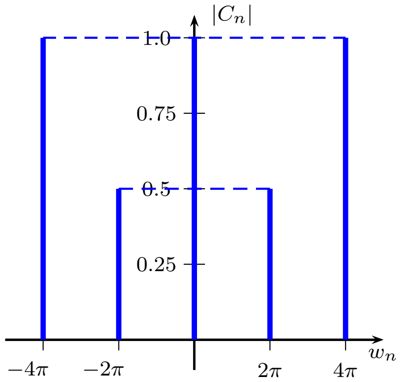

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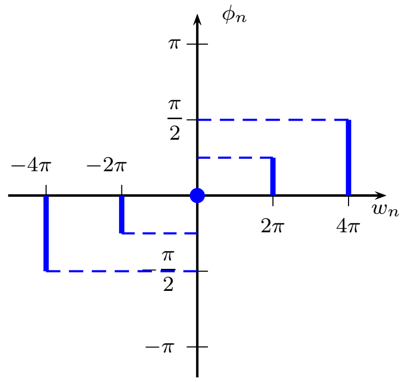

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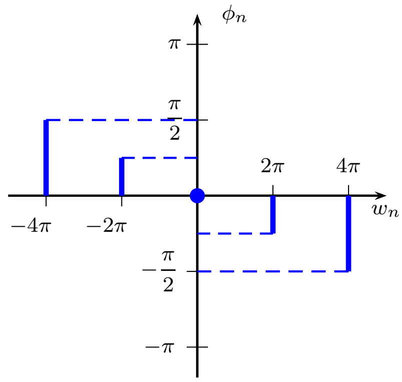

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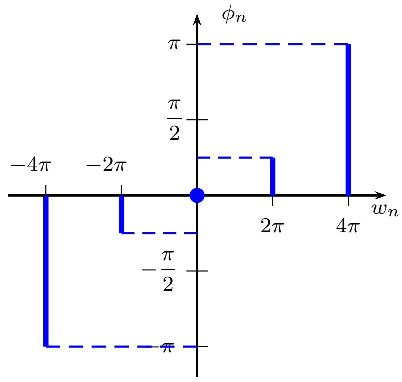

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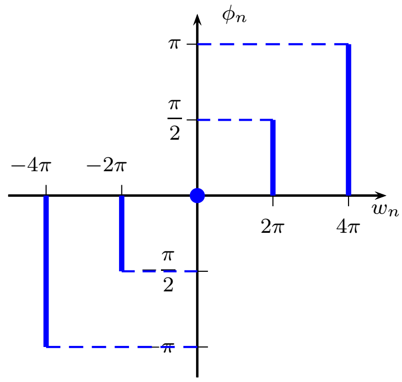

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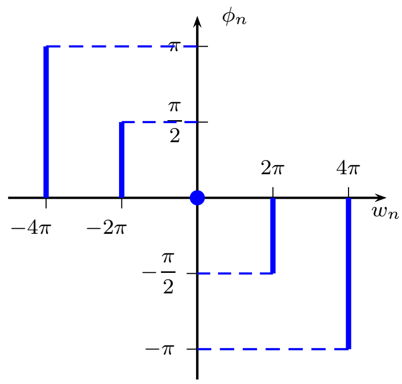
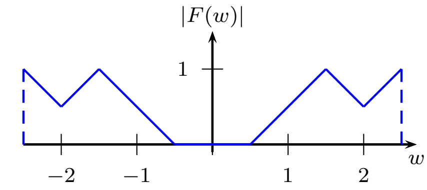


• **Questão 5** (1.0 ponto) Considere o diagrama de espectro de magnitudes da Transformada de Fourier da funç ao $f(t)$ dados nos gráficos abaixo. Assinale na primeira coluna a alternativa que representa o diagrama de espectro de magnitudes de $g(t) = \dfrac{1}{2}f'(t)$ e, na segunda, o diagrama de espectro de magnitudes de $h(t) = 2f(t)\cos\left(\dfrac{5}{2}t\right)$.



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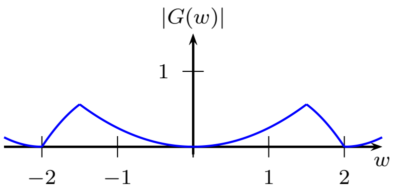

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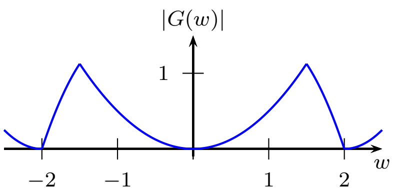

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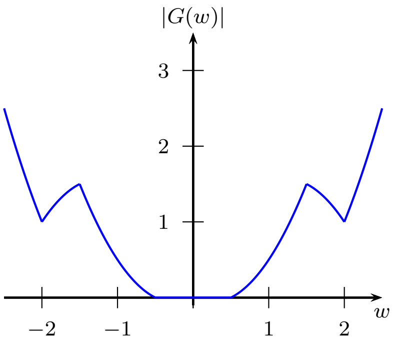

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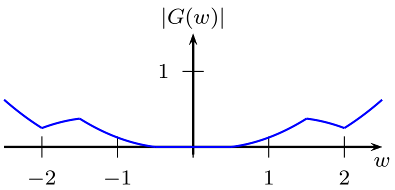

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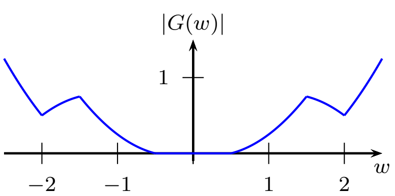

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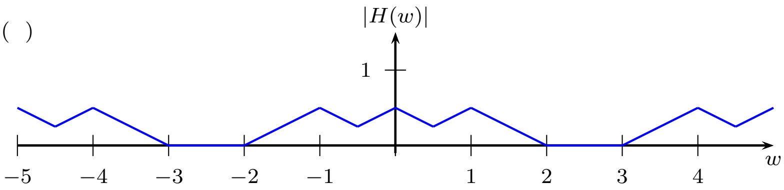

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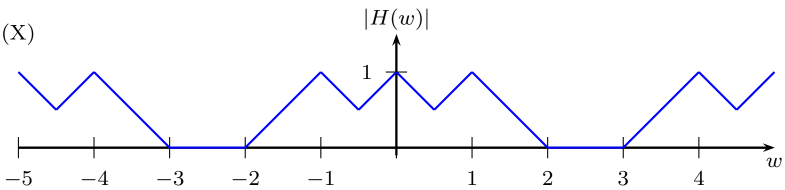

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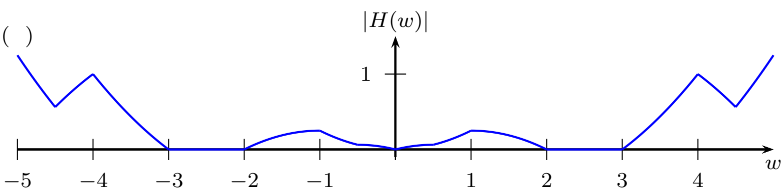

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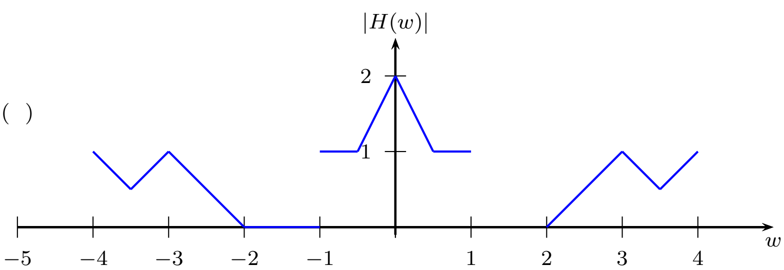

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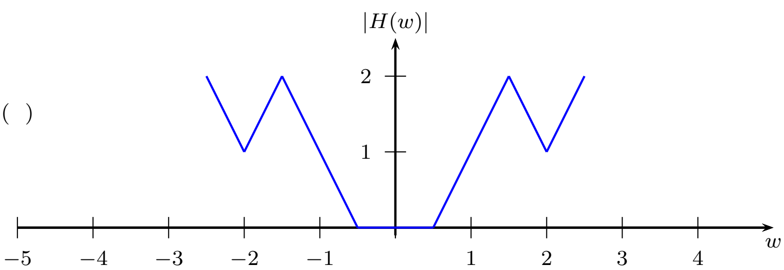


• **Questão 6** (1.0 ponto) Considere

$$f(x) = \begin{cases} 2 & \text{se } 0 < x < 1, \\ 0 & \text{caso contrário.} \end{cases}$$

Assinale na primeira coluna $\mathcal{F}\{f(x)\}$ e, na segunda, $\mathcal{F}\{f(x)\cos(x)\}$.

( ) $\dfrac{2\,\mathrm{sen}(k)}{k}$

( ) $\dfrac{2\,\mathrm{sen}(k)}{k} + i\dfrac{2\cos(k)}{k}$

( ) $\dfrac{2\,\mathrm{sen}(k)}{k} + 2i\left(\dfrac{\cos(k)}{k} - 1\right)$

( ) $\dfrac{2\,\mathrm{sen}(k) - 2}{k} + 2i\dfrac{\cos(k)}{k}$

(X) $\dfrac{2\,\mathrm{sen}(k)}{k} + 2i\dfrac{\cos(k) - 1}{k}$

(X) $\dfrac{\mathrm{sen}(k+1)}{k+1} + \dfrac{\mathrm{sen}(k-1)}{k-1} + i\left(\dfrac{\cos(k+1) - 1}{k+1} + \dfrac{\cos(k-1) - 1}{k-1}\right)$

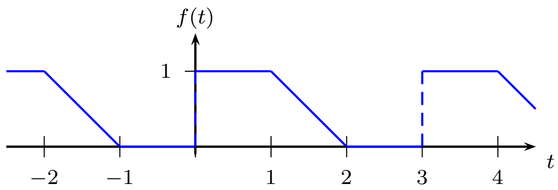
( ) $\dfrac{\mathrm{sen}(k+1)}{k+1} + i\dfrac{\cos(k-1) - 1}{k-1}$

( ) $\dfrac{\mathrm{sen}(k+1)}{k} + \dfrac{\mathrm{sen}(k-1)}{k} + i\left(\dfrac{\cos(k+1) - 1}{k} + \dfrac{\cos(k-1) - 1}{k}\right)$

( ) $\dfrac{\mathrm{sen}(k-1)}{k-1} + i\dfrac{\cos(k+1) - 1}{k+1}$

( ) $\dfrac{\mathrm{sen}(k+1)}{k+1} + \dfrac{\mathrm{sen}(k+1)}{k+1} + i\left(\dfrac{\cos(k-1) - 1}{k-1} + \dfrac{\cos(k-1) - 1}{k-1}\right)$

• **Questão 7** (2.0 ponto) Calcule a série de Fourier para a seguinte função periódica:



Solução: Observamos que $T = 3$ e $w_n = \dfrac{2\pi n}{3}$.

$$a_0 = \frac{2}{3}\int_0^3 f(t)dt = \frac{2}{3}\left(\int_0^1 1dt + \int_1^2 (2-t)dt\right) = \frac{2}{3}\left(1 + \left(2t - \frac{t^2}{2}\right)\Bigg|_1^2\right) = 1$$

$$\begin{aligned}
a_n &= \frac{2}{3}\int_0^3 f(t)\cos(w_n t)dt \\
&= \frac{2}{3}\left(\int_0^1 \cos(w_n t)dt + \int_1^2 (2-t)\cos(w_n t)dt\right) \\
&= \frac{2}{3}\left(\int_0^1 \cos(w_n t)dt + 2\int_1^2 \cos(w_n t)dt - \int_1^2 t\cos(w_n t)dt\right) \\
&= \frac{2}{3}\left(\frac{\operatorname{sen}(w_n t)}{w_n}\Bigg|_0^1 + 2\,\frac{\operatorname{sen}(w_n t)}{w_n}\Bigg|_1^2 - \frac{t\operatorname{sen}(w_n t)}{w_n}\Bigg|_1^2 + \int_1^2 \frac{\operatorname{sen}(w_n t)}{w_n}dt\right) \\
&= \frac{2}{3}\left(\frac{\operatorname{sen}(w_n)}{w_n} + 2\frac{\operatorname{sen}(2w_n)}{w_n} - 2\frac{\operatorname{sen}(w_n)}{w_n} - \frac{2\operatorname{sen}(2w_n)}{w_n} + \frac{\operatorname{sen}(w_n)}{w_n} - \frac{\cos(w_n t)}{w_n^2}\Bigg|_1^2\right) \\
&= \frac{2}{3}\left(\frac{\cos(w_n)}{w_n^2} - \frac{\cos(2w_n)}{w_n^2}\right) \\
&= \frac{2}{3}\left(\frac{9\cos(\frac{2\pi n}{3})}{4\pi^2 n^2} - \frac{9\cos(\frac{4\pi n}{3})}{4\pi^2 n^2}\right) \\
&= \frac{3}{2}\left(\frac{\cos(\frac{2\pi n}{3})}{\pi^2 n^2} - \frac{\cos(\frac{4\pi n}{3})}{\pi^2 n^2}\right)
\end{aligned}$$

$$\begin{aligned}
b_n &= \frac{2}{3}\int_0^3 f(t)\operatorname{sen}(w_n t)dt \\
&= \frac{2}{3}\left(\int_0^1 \operatorname{sen}(w_n t)dt + \int_1^2 (2-t)\operatorname{sen}(w_n t)dt\right) \\
&= \frac{2}{3}\left(\int_0^1 \operatorname{sen}(w_n t)dt + 2\int_1^2 \operatorname{sen}(w_n t)dt - \int_1^2 t\operatorname{sen}(w_n t)dt\right) \\
&= \frac{2}{3}\left(-\frac{\cos(w_n t)}{w_n}\Bigg|_0^1 - 2\,\frac{\cos(w_n t)}{w_n}\Bigg|_1^2 + \frac{t\cos(w_n t)}{w_n}\Bigg|_1^2 - \int_1^2 \frac{\cos(w_n t)}{w_n}dt\right) \\
&= \frac{2}{3}\left(\frac{1-\cos(w_n)}{w_n} + 2\frac{\cos(w_n)}{w_n} - 2\frac{\cos(2w_n)}{w_n} + \frac{2\cos(2w_n)}{w_n} - \frac{\cos(w_n)}{w_n} - \frac{\operatorname{sen}(w_n t)}{w_n^2}\Bigg|_1^2\right) \\
&= \frac{2}{3}\left(\frac{1}{w_n} + \frac{\operatorname{sen}(w_n)}{w_n^2} - \frac{\operatorname{sen}(2w_n)}{w_n^2}\right) \\
&= \frac{2}{3}\left(\frac{3}{2\pi n} + \frac{9\operatorname{sen}(\frac{2\pi n}{3})}{4\pi^2 n^2} - \frac{9\operatorname{sen}(\frac{4\pi n}{3})}{4\pi^2 n^2}\right) \\
&= \frac{1}{\pi n} + \frac{3\operatorname{sen}(\frac{2\pi n}{3})}{2\pi^2 n^2} - \frac{3\operatorname{sen}(\frac{4\pi n}{3})}{2\pi^2 n^2}.
\end{aligned}$$

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$
a_n	1	0	0	0	0	0	0	0	0
b_n	/	$\frac{1}{\pi} + \frac{3\sqrt{3}}{2\pi^2}$	$\frac{1}{2\pi} - \frac{3\sqrt{3}}{8\pi^2}$	$\frac{1}{3\pi}$	$\frac{1}{4\pi} + \frac{3\sqrt{3}}{32\pi^2}$	$\frac{1}{5\pi} - \frac{3\sqrt{3}}{50\pi^2}$	$\frac{1}{6\pi}$	$\frac{1}{7\pi} + \frac{3\sqrt{3}}{98\pi^2}$	$\frac{1}{8\pi} - \frac{3\sqrt{3}}{128\pi^2}$

• **Questão 8** (2.0 pontos) Resolva o seguinte problema de difusão de calor com velocidade:

$$u_t + 3u_x - u_{xx} = 0$$
$$u(x,0) = 120\delta(x).$$

Solução: Usamos a notação $\mathcal{F}\{u(x,t)\} = U(k,t)$ e aplicamos a Transformada de Fourier para obter

$$\begin{aligned} U_t &= -3ikU - k^2U \\ U(k,0) &= 120. \end{aligned}$$

A solução da equação acima é calculada por separação de variáveis:

$$U(k,t) = 120e^{-3ik-k^2} = 120e^{-3ikt}e^{-k^2t}.$$

Calculamos a transformada inversa da seguinte forma:

i) Transformada inversa da função e^{-k^2t}:

$$\begin{aligned} \mathcal{F}\left\{e^{-k^2t}\right\} &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-k^2t}e^{ikx}dk \\ &= \frac{1}{\pi}\int_0^{\infty} e^{-k^2t}\cos(kx)dk = \frac{1}{\sqrt{4\pi t}}e^{-\frac{x^2}{4t}} \end{aligned}$$

ii) Transformada inversa da função $120e^{-3ik}e^{-k^2}$ usando a propriedade do deslocamento:

$$u(x,t) = \mathcal{F}\left\{120e^{-3ikt}e^{-k^2t}\right\} = \frac{60}{\sqrt{\pi t}}e^{-\frac{(x-3t)^2}{4t}}$$