UFRGS - INSTITUTO DE MATEMÁTICA E ESTATÍSTICA
Departamento de Matemática Pura e Aplicada
MAT01168 - Turma C - 2017/1
Prova da área IIA

1 - 5	6	7	Total

Nome: ______________________________ **Cartão:** ______________

Regras Gerais:

- Não é permitido o uso de calculadoras, telefones ou qualquer outro recurso computacional ou de comunicação.
- Trabalhe individualmente e sem uso de material de consulta além do fornecido.
- Devolva o caderno de questões preenchido ao final da prova.

Regras para as questões abertas:

- Seja sucinto, completo e claro.
- Justifique todo procedimento usado.
- Indique identidades matemáticas usadas, em especial, itens da tabela.
- Use notação matemática consistente.

Identidades:

$\operatorname{sen}(x) = \dfrac{e^{ix} - e^{-ix}}{2i}$	$\cos(x) = \dfrac{e^{ix} + e^{-ix}}{2}$
$\operatorname{senh}(x) = \dfrac{e^{x} - e^{-x}}{2}$	$\cosh(x) = \dfrac{e^{x} + e^{-x}}{2}$
$(a+b)^n = \sum_{j=0}^{\infty} \binom{n}{j} a^{n-j} b^j, \quad \binom{n}{j} = \dfrac{n!}{j!(n-j)!}$	
$\operatorname{sen}(x+y) = \operatorname{sen}(x)\cos(y) + \operatorname{sen}(y)\cos(x)$	
$\cos(x+y) = \cos(x)\cos(y) - \operatorname{sen}(x)\operatorname{sen}(y)$	

Propriedades:

1	Linearidade	$\mathcal{L}\{\alpha f(t) + \beta g(t)\} = \alpha \mathcal{L}\{f(t)\} + \beta \mathcal{L}\{g(t)\}$
2	Transformada da derivada	$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$ $\mathcal{L}\{f''(t)\} = s^2\mathcal{L}\{f(t)\} - sf(0) - f'(0)$
3	Deslocamento no eixo s	$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$
4	Deslocamento no eixo t	$\mathcal{L}\{u(t-a)f(t-a)\} = e^{-as}F(s)$ $\mathcal{L}\{u(t-a)\} = \dfrac{e^{-as}}{s}$
5	Transformada da integral	$\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \dfrac{F(s)}{s}$
6	Filtragem da Delta de Dirac	$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$
7	Transformada da Delta de Dirac	$\mathcal{L}\{\delta(t-a)\} = e^{-as}$
8	Teorema da Convolução	$\mathcal{L}\{(f*g)(t)\} = F(s)G(s)$, onde $\quad (f*g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$
9	Transformada de funções periódicas	$\mathcal{L}\{f(t)\} = \dfrac{1}{1-e^{-sT}} \int_0^T e^{-s\tau} f(\tau)d\tau$
10	Derivada da transformada	$\mathcal{L}\{tf(t)\} = -\dfrac{dF(s)}{ds}$
11	Integral da transformada	$\mathcal{L}\left\{\dfrac{f(t)}{t}\right\} = \int_s^{\infty} F(\hat{s})\hat{s}$

Séries:

$\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 \cdots, \quad -1 < x < 1$
$\dfrac{x}{(1-x)^2} = \sum_{n=1}^{\infty} n x^n = x + 2x^2 + 3x^3 + \cdots, \quad -1 < x < 1$
$e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots, \quad -\infty < x < \infty$
$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{n+1}}{n+1}, \quad -1 < x < 1$
$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1}, \quad -1 < x < 1$
$\operatorname{sen}(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!}, \quad -\infty < x < \infty$
$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!}, \quad -\infty < x < \infty$
$\operatorname{senh}(x) = \sum_{n=0}^{\infty} \dfrac{x^{2n+1}}{(2n+1)!}, \quad -\infty < x < \infty$
$\cosh(x) = \sum_{n=0}^{\infty} \dfrac{x^{2n}}{(2n)!}, \quad -\infty < x < \infty$
$(1+x)^m = 1 + \sum_{n=1}^{\infty} \dfrac{m(m-1)\cdots(m-n+1)}{n!} x^n,$ $-1 < x < 1,\ m \neq 0, 1, 2, ...$

Funções especiais:

Função Gamma	$\Gamma(k) = \int_0^{\infty} x^{k-1} e^{-x} dx$
Propriedade da Função Gamma	$\Gamma(k+1) = k\Gamma(k), \quad k > 0$ $\Gamma(n+1) = n!, \quad n \in \mathbb{N}$
Função de Bessel modificada de ordem ν	$I_\nu(x) = \sum_{m=0}^{\infty} \dfrac{1}{m!\Gamma(m+\nu+1)} \left(\dfrac{x}{2}\right)^{2m+\nu}$
Função de Bessel de ordem 0	$J_0(x) = \sum_{m=0}^{\infty} \dfrac{(-1)^m}{m!^2} \left(\dfrac{x}{2}\right)^{2m}$
Integral seno	$\operatorname{Si}(t) = \int_0^t \dfrac{\operatorname{sen}(x)}{x} dx$

Integrais:

$\int x e^{\lambda x}\, \mathrm{d}x = \dfrac{e^{\lambda x}}{\lambda^2}(\lambda x - 1) + C$
$\int x^2 e^{\lambda x}\, \mathrm{d}x = e^{\lambda x}\left(\dfrac{x^2}{\lambda} - \dfrac{2x}{\lambda^2} + \dfrac{2}{\lambda^3}\right) + C$
$\int x^n e^{\lambda x}\, \mathrm{d}x = \dfrac{1}{\lambda} x^n e^{\lambda x} - \dfrac{n}{\lambda} \int x^{n-1} e^{\lambda x} \mathrm{d}x + C$
$\int x \cos(\lambda x)\, dx = \dfrac{\cos(\lambda x) + \lambda x \operatorname{sen}(\lambda x)}{\lambda^2} + C$
$\int x \operatorname{sen}(\lambda x)\, dx = \dfrac{\operatorname{sen}(\lambda x) - \lambda x \cos(\lambda x)}{\lambda^2} + C$

Tabela de transformadas de Laplace:

	$F(s)=\mathcal{L}\{f(t)\}$	$f(t)=\mathcal{L}^{-1}\{F(s)\}$
1	$\frac{1}{s}$	1
2	$\frac{1}{s^2}$	t
3	$\frac{1}{s^n}, \quad (n=1,2,3,...)$	$\frac{t^{n-1}}{(n-1)!}$
4	$\frac{1}{\sqrt{s}},$	$\frac{1}{\sqrt{\pi t}}$
5	$\frac{1}{s^{\frac{3}{2}}},$	$2\sqrt{\frac{t}{\pi}}$
6	$\frac{1}{s^k}, \quad (k>0)$	$\frac{t^{k-1}}{\Gamma(k)}$
7	$\frac{1}{s-a}$	e^{at}
8	$\frac{1}{(s-a)^2}$	te^{at}
9	$\frac{1}{(s-a)^n}, \quad (n=1,2,3...)$	$\frac{1}{(n-1)!}t^{n-1}e^{at}$
10	$\frac{1}{(s-a)^k}, \quad (k>0)$	$\frac{1}{\Gamma(k)}t^{k-1}e^{at}$
11	$\frac{1}{(s-a)(s-b)}, \quad (a\neq b)$	$\frac{1}{a-b}\left(e^{at}-e^{bt}\right)$
12	$\frac{s}{(s-a)(s-b)}, \quad (a\neq b)$	$\frac{1}{a-b}\left(ae^{at}-be^{bt}\right)$
13	$\frac{1}{s^2+w^2}$	$\frac{1}{w}\operatorname{sen}(wt)$
14	$\frac{s}{s^2+w^2}$	$\cos(wt)$
15	$\frac{1}{s^2-a^2}$	$\frac{1}{a}\operatorname{senh}(at)$
16	$\frac{s}{s^2-a^2}$	$\cosh(at)$
17	$\frac{1}{(s-a)^2+w^2}$	$\frac{1}{w}e^{at}\operatorname{sen}(wt)$
18	$\frac{s-a}{(s-a)^2+w^2}$	$e^{at}\cos(wt)$
19	$\frac{1}{s(s^2+w^2)}$	$\frac{1}{w^2}(1-\cos(wt))$
20	$\frac{1}{s^2(s^2+w^2)}$	$\frac{1}{w^3}(wt-\operatorname{sen}(wt))$
21	$\frac{1}{(s^2+w^2)^2}$	$\frac{1}{2w^3}(\operatorname{sen}(wt)-wt\cos(wt))$
22	$\frac{s}{(s^2+w^2)^2}$	$\frac{t}{2w}\operatorname{sen}(wt)$
23	$\frac{s^2}{(s^2+w^2)^2}$	$\frac{1}{2w}(\operatorname{sen}(wt)+wt\cos(wt))$
24	$\frac{s}{(s^2+a^2)(s^2+b^2)}, \quad (a^2\neq b^2)$	$\frac{1}{b^2-a^2}(\cos(at)-\cos(bt))$
25	$\frac{1}{(s^4+4a^4)}$	$\frac{1}{4a^3}[\operatorname{sen}(at)\cosh(at)- -\cos(at)\operatorname{senh}(at)]$
26	$\frac{s}{(s^4+4a^4)}$	$\frac{1}{2a^2}\operatorname{sen}(at)\operatorname{senh}(at))$
27	$\frac{1}{(s^4-a^2)}$	$\frac{1}{2a^3}(\operatorname{senh}(at)-\operatorname{sen}(at))$
28	$\frac{s}{(s^4-a^4)}$	$\frac{1}{2a^2}(\cosh(at)-\cos(at))$
29	$\sqrt{s-a}-\sqrt{s-b}$	$\frac{1}{2\sqrt{\pi t^3}}(e^{bt}-e^{at})$
30	$\frac{1}{\sqrt{s+a}\sqrt{s+b}}$	$e^{\frac{-(a+b)t}{2}}I_0\left(\frac{a-b}{2}t\right)$
31	$\frac{1}{\sqrt{s^2+a^2}}$	$J_0(at)$
32	$\frac{s}{(s-a)^{\frac{3}{2}}}$	$\frac{1}{\sqrt{\pi t}}e^{at}(1+2at)$
33	$\frac{1}{(s^2-a^2)^k}, \quad (k>0)$	$\frac{\sqrt{\pi}}{\Gamma(k)}\left(\frac{t}{2a}\right)^{k-\frac{1}{2}}I_{k-\frac{1}{2}}(at)$
34	$\frac{1}{s}e^{-\frac{k}{s}}, \quad (k>0)$	$J_0(2\sqrt{kt})$
35	$\frac{1}{\sqrt{s}}e^{-\frac{k}{s}}$	$\frac{1}{\sqrt{\pi t}}\cos(2\sqrt{kt})$
36	$\frac{1}{s^{\frac{3}{2}}}e^{\frac{k}{s}}$	$\frac{1}{\sqrt{\pi t}}\operatorname{senh}(2\sqrt{kt})$
37	$e^{-k\sqrt{s}}, \quad (k>0)$	$\frac{k}{2\sqrt{\pi t^3}}e^{-\frac{k^2}{4t}}$
38	$\frac{1}{s}\ln(s)$	$-\ln(t)-\gamma, \quad (\gamma\approx 0,5772)$
39	$\ln\left(\frac{s-a}{s-b}\right)$	$\frac{1}{t}\left(e^{bt}-e^{at}\right)$
40	$\ln\left(\frac{s^2+w^2}{s^2}\right)$	$\frac{2}{t}(1-\cos(wt))$
41	$\ln\left(\frac{s^2-a^2}{s^2}\right)$	$\frac{2}{t}(1-\cosh(at))$
42	$\tan^{-1}\left(\frac{w}{s}\right)$	$\frac{1}{t}\operatorname{sen}(wt)$
43	$\frac{1}{s}\cot^{-1}(s)$	$\operatorname{Si}(t)$
44	$\frac{1}{s}\tanh\left(\frac{as}{2}\right)$	Onda quadrada $f(t)=\begin{cases}1, & 0<t<a\\ -1, & a<t<2a\end{cases}$ $f(t+2a)=f(t), \quad t>0$
45	$\frac{1}{as^2}\tanh\left(\frac{as}{2}\right)$	Onda triangular $f(t)=\begin{cases}\frac{t}{a}, & 0<t<a\\ -\frac{t}{a}+2, & a<t<2a\end{cases}$ $f(t+2a)=f(t), \quad t>0$
46	$\frac{w}{(s^2+w^2)\left(1-e^{-\frac{\pi}{w}s}\right)}$	Retificador de meia onda $f(t)=\begin{cases}\operatorname{sen}(wt), & 0<t<\frac{\pi}{w}\\ 0, & \frac{\pi}{w}<t<\frac{2\pi}{w}\end{cases}$ $f\left(t+\frac{2\pi}{w}\right)=f(t), \quad t>0$
47	$\frac{w}{s^2+w^2}\coth\left(\frac{\pi s}{2w}\right)$	Retificador de onda completa $f(t)=\lvert\operatorname{sen}(wt)\rvert$
48	$\frac{1}{as^2}-\frac{e^{-as}}{s\,(1-e^{-as})}$	Onda dente de serra $f(t)=\frac{t}{a}, \quad 0<t<a$ $f(t)=f(t-a), \quad t>a$

• **Questão 1** (1.0 ponto) Seja $f(t) = u(t-1)(1-u(t-3))$. Assinale as alternativas que indicam respectivamente $\mathcal{L}\{f(t)\}$ e $\mathcal{L}\{tf(t)\}$

( ) $\frac{1}{s^2}\left(e^{-s} - e^{-4s}\right)$,

( ) $\frac{1}{s^2}\left(e^{-s} - e^{-3s}\right)$,

( ) $\frac{1}{s}\left(e^{-s} - e^{-4s}\right)$,

( ) $\frac{1}{s}\left(e^{-s} - e^{-3s}\right)$,

( ) n.d.a.

( ) $\frac{1}{s^4}\left(e^{-s} - e^{-3s}\right) + \frac{1}{s}\left(e^{-s} - 3e^{-3s}\right)$,

( ) $\frac{1}{s^4}\left(e^{-s} - e^{-3s}\right) + \frac{1}{s}\left(e^{-s} - 4e^{-4s}\right)$,

( ) $\frac{1}{s^2}\left(e^{-s} - e^{-4s}\right) + \frac{1}{s}\left(e^{-s} - 4e^{-4s}\right)$,

( ) $\frac{1}{s^2}\left(e^{-s} - e^{-3s}\right) + \frac{1}{s}\left(e^{-s} - 3e^{-3s}\right)$,

( ) n.d.a.

• **Questão 2** (1.0 ponto) Considere a função $f(t) = (t-1)^2 u(t-2)$. Assinale as alternativas que indicam, respectivamente, $\mathcal{L}\{f(t)\}$ e $\mathcal{L}\{f'(t)\}$:

( ) $\mathcal{L}\{f(t)\} = \frac{2}{s^2}e^{-2s}$

( ) $\mathcal{L}\{f(t)\} = \left(\frac{2}{s^2} + \frac{2}{s} + 1\right)e^{-2s}$

( ) $\mathcal{L}\{f(t)\} = \frac{2}{s^3}e^{-2s}$

( ) $\mathcal{L}\{f(t)\} = \left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right)e^{-2s}$

( ) $\mathcal{L}\{f(t)\} = \left(\frac{2}{s^3} + \frac{2}{s^2}\right)e^{-2s}$

( ) n.d.a.

( ) $\mathcal{L}\{f'(t)\} = \left(\frac{2}{s^2} + \frac{2}{s}\right)e^{-2s}$

( ) $\mathcal{L}\{f'(t)\} = \frac{2}{s^2}e^{-2s}$

( ) $\mathcal{L}\{f'(t)\} = \frac{2}{s}e^{-2s}$

( ) $\mathcal{L}\{f'(t)\} = \left(\frac{2}{s^2} + \frac{2}{s} + s\right)e^{-2s}$

( ) $\mathcal{L}\{f'(t)\} = \left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right)e^{-2s}$

( ) $\mathcal{L}\{f'(t)\} = \left(\frac{2}{s^2} + \frac{2}{s} + 1\right)e^{-2s}$

( ) n.d.a.

• **Questão 3** (1.0) Considere a equação integral $y(t) = 1 - \int_0^t y(\tau)e^{t-\tau}d\tau$. Assinale as alternativas que indicam, respectivamente, $\mathcal{L}\{y(t)\}$ e $\mathcal{L}\left\{e^{-t}y(t)\right\}$.

( ) $\frac{s-1}{s^3}$,

( ) $\frac{1}{s+1}$,

( ) $\frac{1}{s^2+1}$,

( ) $\frac{1-s}{s^2}$,

( ) $\frac{s-1}{s^2}$,

( ) n.d.a.

( ) $\frac{1}{s}$,

( ) $\frac{s}{(s+1)^2}$,

( ) $\frac{1}{(s-1)^2+1}$,

( ) $\frac{s}{(s+1)^3}$,

( ) $\frac{s-1}{s^2}$

( ) n.d.a.

• **Questão 4** (1.0) Considere a função $f(t) = tu(t) - 2(t-1)u(t-1) + (t-2)u(t-3)$. Assinale as alternativas que indicam o esboço do gráfico de f e f', respectivamente.

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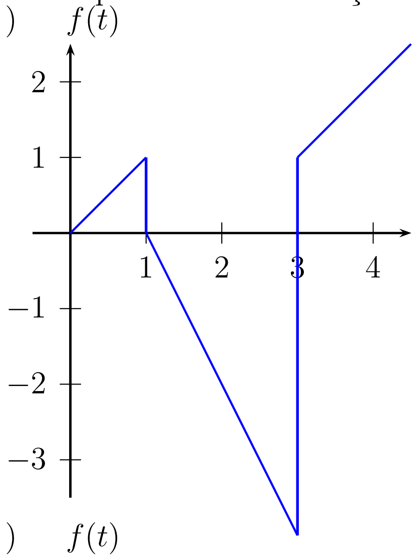

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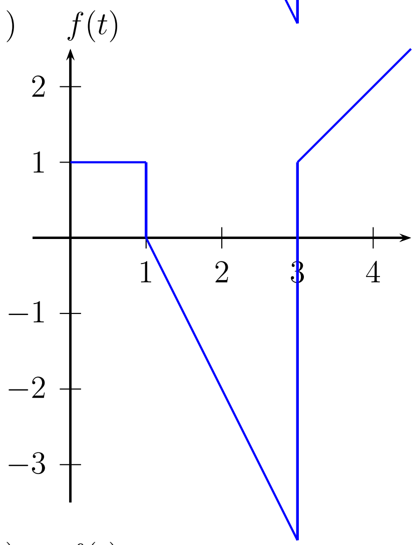

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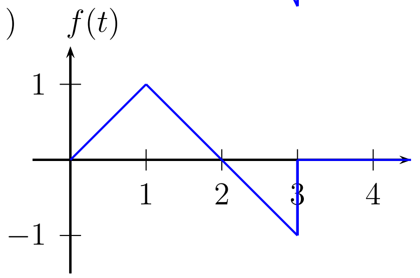

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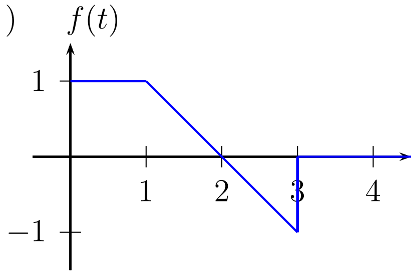

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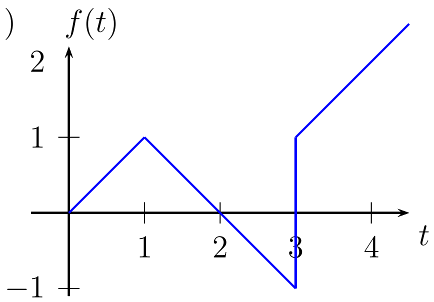

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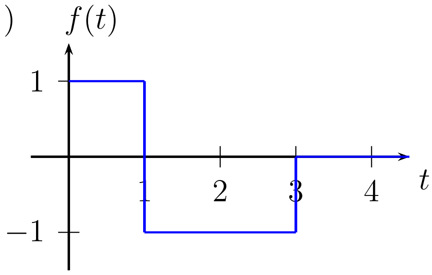

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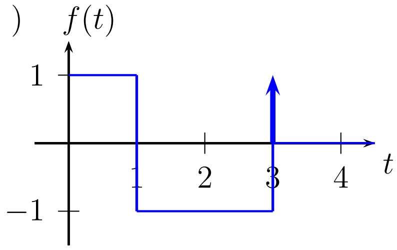

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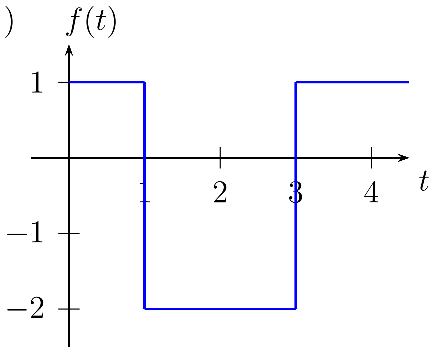

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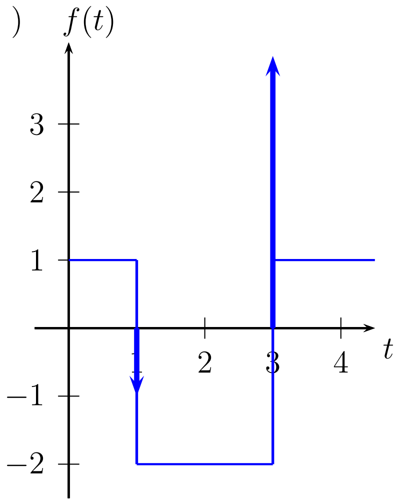

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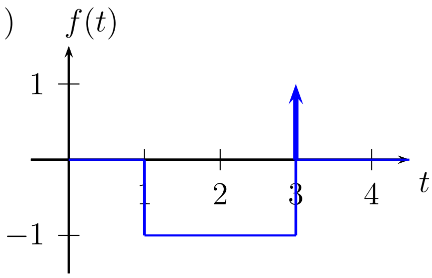
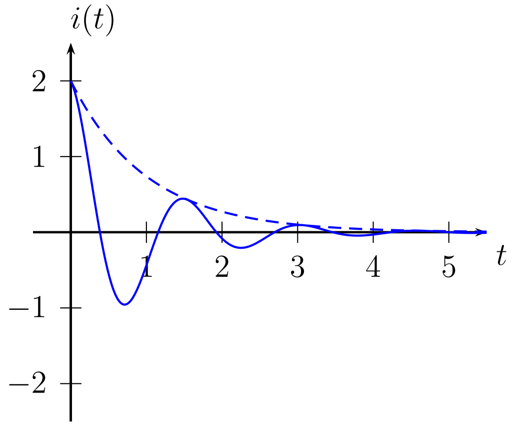


• **Questão 5** (1.0) Considere um sistema RLC modelado pelo problema de segunda ordem abaixo e o gráfico da corrente esboçado ao lado.

$$\begin{aligned} Li''(t) + Ri'(t) + \frac{i(t)}{C} &= f(t) \\ i(0) &= i_0 \\ i'(0) &= i'_0 \end{aligned}$$



A equação da envoltória, esboçada com uma linha pontilhada, é $2e^{-t}$. Assinale as alternativas que indicam, respectivamente, a frequência do sinal e a transformada de Laplace $I(s) = \{i(t)\}$.

( ) $f = \dfrac{1}{3} Hz$

( ) $f = \dfrac{2}{3} Hz$

( ) $f = \dfrac{4}{3} Hz$

( ) $f = \dfrac{\pi}{3} Hz$

( ) $f = \dfrac{2\pi}{3} Hz$

( ) $f = \dfrac{4\pi}{3} Hz$

( ) $I(s) = \dfrac{(s-1)}{(s-1)^2 + \left(\frac{2\pi}{3}\right)^2}$

( ) $I(s) = \dfrac{2(s+1)}{(s+1)^2 - \left(\frac{\pi}{3}\right)^2}$

( ) $I(s) = \dfrac{2(s+1)}{(s+1)^2 + \left(\frac{4\pi}{3}\right)^2}$

( ) $I(s) = \dfrac{2(s-1)}{(s-1)^2 + \left(\frac{2\pi}{3}\right)^2}$

( ) $I(s) = \dfrac{2}{(s+1)^2 + \left(\frac{\pi}{3}\right)^2}$

( ) $I(s) = \dfrac{3}{2\pi} \dfrac{(s+1)}{(s+1)^2 + \left(\frac{4\pi}{3}\right)^2}$

• **Questão 6** (2.5) Considere o seguinte problema de valor inicial:

$$\begin{aligned} x'(t) &= -x(t) + y(t) + \delta(t) \\ y(t) &= -\int_0^t e^{(t-\tau)} y(\tau) d\tau + x(t), \end{aligned}$$

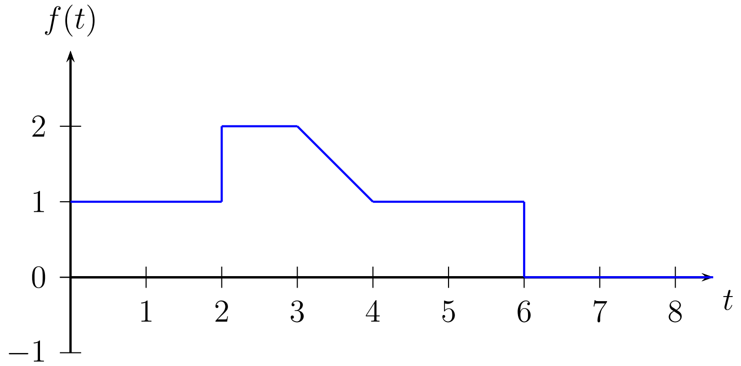
com $x(0) = 0$.

a) (0.9) Aplique o método de transformada de Laplace e escreva $X(s) = \mathcal{L}\{x(t)\}$ e $Y(s) = \mathcal{L}\{y(t)\}$ nos espaços abaixo.

b) (0.9) Calcule as transformadas inversas e escreva $x(t)$ e $y(t)$ abaixo.

c) (0.7) Aplique sua solução do item b) nas condições inciais e justifique o resultado encontrado.

- **Questão 7** (2.5) Considere a função dada no gráfico abaixo



a) (0.8 ponto) Escreve em termos de Delta de Dirac e Heaviside as funções f, f' e $g(t) = \displaystyle\int_0^t f(\tau)d\tau$.

b) (0.8 ponto) Esboce o gráfico de $f'(t)$ e $g(t) = \displaystyle\int_0^t f(\tau)d\tau$ indicando eixos e valores notáveis.

c) (0.9 ponto) Calcule as transformadas de Laplace das funções f, f' e $g(t) = \displaystyle\int_0^t f(\tau)d\tau$.